Geometric phases induced in auxiliary qubits by many-body systems near its critical points

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(Dated: February 9, 2008)

The geometric phase induced in an auxiliary qubit by a many-body system is calculated and discussed. Two kinds of coupling between the auxiliary qubit and the many-body system are considered, which lead to dephasing and dissipation in the qubit, respectively. As an example, we consider the XY spin-chain dephasingly couple to a qubit, the geometric phase induced in the qubit is presented and discussed. The results show that the geometric phase might be used to signal the critical points of the many-body system, and it tends to zero with the parameters of the many-body system going away from the critical points.

PACS numbers: 03.65.Ud, 03.65.Bz

I. INTRODUCTION

Motivated by the fact that all realistic quantum systems are coupled to their surrounding environments, many researchers have paid their attention on the study of geometric phase in open systems since 1980s[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. From the perspective of application, the use of geometric phase in the implementation of fault-tolerant quantum gates [16, 17, 18, 19] also requires the study of geometric phase in open systems, because the system which needs to be manipulated may decoher from a quantum superposition into statistical mixtures due to coupling to its environment. Despite of extensive studies that have been performed in this direction, investigation into the effect of correlation among the environmental systems on the induced geometric phase in open systems is lacking.

Geometric phases have been shown to associate with a variety of condensed matter phenomena [20, 21, 22, 23]. Nevertheless, their connection to quantum phase transitions has only been given recently in [24, 25, 26], where it has been shown that the Berry's phase can be used to signal the critical points of the spin chain[24], the critical exponents were evaluated from the scaling behavior of geometric phases[25], and the geometric phase can be considered as a topological test to reveal quantum phase transitions[26]. In these works, the geometric phase analyzed is acquired by the ground state (or the low lying excited states) of the many-body system. This gives rise to the following question, whether the geometric phase induced in an auxiliary system by the many-body system has connection to the critical points of the many-body system. In this paper, we shall try to answer this question by examining the geometric phase induced in a qubit by the many-body system. This makes our study different from that in [24, 25, 26]. Two kinds of coupling between the auxiliary qubit and the many-body system are considered. The first kind of coupling commutes with the free Hamiltonian of the auxiliary qubit, leading to dephasing in the qubit. Whereas the second one commutes with the free Hamiltonian of the many-body system, which causes energy loss (dissipation) of the qubit.

This paper is organized as follows. In Sec.II we present a general formulism to calculate and analyze the reduced density matrix of the auxiliary qubit, and establish a connection of the reduced density matrix to the critical points. As an example, we calculate the geometric phase induced in the auxiliary qubit taking the XY spin chain as the many-body system in Sec.III. And finally we conclude our results in Sec.IV.

II. GENERAL

In this section, we will give a general formalism to analyze the reduced density matrix of a qubit coupled to a quantum many-body system, and establish a connection between the reduced density matrix of the qubit and the criticality in the quantum system. We restrict ourself to consider the following qubit to many-body system couplings. First we consider the case where the coupling conserves the energy of the qubit. Then we analyze the case where the energy of the qubit does not conserve, but the energy of the quantum many-body system conserves. The first case corresponds to dephasing in the qubit, while the second kind of coupling results in dissipation in the qubit.

Consider a qubit coupled to a quantum many-body system. The Hamiltonian that governs the evolution of the whole system may have the form

$$H = H_{\frac{1}{2}} + H_m(\lambda) + H_i, \tag{1}$$

where $H_{\frac{1}{2}} = \mu|\uparrow\rangle\langle\uparrow|$, describes the free Hamiltonian of the qubit, $H_m(\lambda)$ stands for the free Hamiltonian of the quantum many-body system, and $H_i = gH_M \otimes |\uparrow\rangle\langle\uparrow|$ represents the coupling between them. H_M is an arbitrary operator of the many-body system. It is clear that $[H_i, H_{\frac{1}{2}}] = 0$, therefore the energy of the qubit conserves. The quantum system described by $H_m(\lambda)$ undergoes a quantum phase transition for parameter $\lambda = \lambda_c$. It is

easy to show that the time evolution operator for the whole system may be written as

$$U(t) = U_{\uparrow}(t)|\uparrow\rangle\langle\uparrow| + U_{\downarrow}(t)|\downarrow\rangle\langle\downarrow|, \quad (2)$$

with $U_{\uparrow}(t)$ and $U_{\downarrow}(t)$ satisfying

$$i\hbar\frac{\partial}{\partial t}U_{\uparrow,\downarrow}(t) = H_{\uparrow,\downarrow}U_{\uparrow,\downarrow}(t), \quad (3)$$

where

$$H_{\uparrow/\downarrow} = H_m + (\frac{\mu}{2} \pm \frac{\mu}{2}) + (\frac{g}{2} \pm \frac{g}{2})H_M. \quad (4)$$

$|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of σ^z for the qubit. Having these expressions, we now show that the off-diagonal elements of the reduced density matrix of the qubit change drastically in the vicinity of critical points. To this end, we assume that the qubit and the quantum many-body system are initially independent, such that we take the following product state as the initial state of the whole system

$$|\psi(0)\rangle = |\psi_{\frac{1}{2}}(0)\rangle \otimes |G_m\rangle, \quad (5)$$

where $|G_m\rangle\rangle$ represent the ground state of H_m and $|\psi_{\frac{1}{2}}(0)\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$. By a standard calculation, one obtains the reduced density matrix of the qubit

$$\rho_{\frac{1}{2}} = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta F(t) \\ \cos\theta\sin\theta F^*(t) & \sin^2\theta \end{pmatrix}. \quad (6)$$

Here, $F(t)$ is defined by

$$F(t) = \langle G_m|e^{-\frac{i}{\hbar}H_{\uparrow}t}|G_m\rangle, \quad (7)$$

this is the survival probability of the ground state of the quantum system under the action of the Hamiltonian $H_{\uparrow}$. The leading term $F(t) \simeq \langle G_m|G_m^{\uparrow}\rangle$ of this equation represents the overlap function between two ground states $|G_m\rangle$ and $|G_m^{\uparrow}\rangle$, where $|G_m^{\uparrow}\rangle$ denotes the ground state of $H_{\uparrow}$. This overlapping function was shown [27] to take extremal values in the vicinity of critical points. In fact, $F(t)$ represents the Loschmidt echo which goes exponentially to zero as the many-body system approaches the regions of criticality[28]. This give us the behavior of $F(t)$ in the vicinity of critical points λ_c as $F(t) \sim |\lambda - \lambda_c|^{\nu}$, leading to a dramatic change in the induced geometric phase near this point λ_c. We would like to notice that the above discussions hold for a general coupling between the spin and the quantum system, provided the coupling is weak.

Next we turn to study the case when the energy of the qubit does not conserve, but the energy of the many-body system conserves instead. This implies that $[H_{\frac{1}{2}}, H_i] \neq 0$, but $[H_i, H_m] = 0$. Without loss of generality, we consider the following Hamiltonian

$$\begin{aligned} H &= H_{\frac{1}{2}} + H_m(\lambda) + H_i, \\ H_{\frac{1}{2}} &= \Delta\sigma_x, \quad H_i = (g_z\sigma_z + g_y\sigma_y) \otimes H_m(\lambda). \end{aligned} \quad (8)$$

$\sigma_i (i = x, y, z)$ are Pauli operators of the qubit. The interaction Hamiltonian H_i can be rewritten as

$$H_i = ge^{i\gamma\sigma_x}\sigma_z e^{-i\gamma\sigma_x}, \quad (9)$$

with $\cos(2\gamma) = g_z/g$, and $g = \sqrt{g_z^2 + g_z^2}$. The fact that the interaction Hamiltonian H_i commutes with H_m enables us to write the time evolution operator of the whole system as

$$U(t) = \sum_{n=1}^{N} U_n(t)|E_n(\lambda)\rangle\langle E_n(\lambda)|, \quad (10)$$

where $\{|E_n(\lambda)\rangle\}$ are eigenstates of H_m with corresponding eigenenergies $\{E_n = E_n(\lambda)\}$, and we assume that they are nondegenerate. It is easy to show that

$$\begin{aligned} U_n(t) &= e^{-iE_nt}\{\cos\frac{\theta_n}{2}\cos\Omega_n t - i\sigma_x\cos\frac{\theta_n}{2}\sin\Omega_n t \\ &+ i\sin\frac{\theta_n}{2}\cos\Omega_n t[(\cos 2\gamma + \sin 2\gamma)\sigma_z \\ &+ (\sin 2\gamma - \cos 2\gamma)\sigma_y]\}. \end{aligned} \quad (11)$$

Here, $\cos\theta_n = \frac{\Delta}{\sqrt{g^2E_n^2 + \Delta^2}}$, and $\Omega_n = \sqrt{g^2E_n^2 + \Delta^2}$. Quantum phase transition theory tells us that the ground state energy $E_0(\lambda)$ behaves drastically in the vicinity of the critical point λ_c, this would reflect in $U_0(t)$ that is a function of $E_0(\lambda)$. Having established this linkage, we claim that the final state of the qubit

$$\rho_{\frac{1}{2}}(t) = \sum_n |c_n|^2 U_n(t)\rho_{\frac{1}{2}}(0)U_n^{\dagger}(t), \quad (12)$$

and the geometric phase acquired by this state can signal quantum critical points in the many-body system, provided the initial state of the many-body system is a superposition of $\{|E_n\rangle\}$, i.e., $|\psi_m(0)\rangle = \sum_{n=0}^{N} c_n|E_n\rangle$ ($c_0 \neq 0$), and at least two c_n including c_0 are not zero. We would like to emphasize that the geometric phase in this case can not signal the critical points, if the many-body system is initially in the ground state with probability 1. This is because the qubit-system interaction could not excite the many-body system in this situation. In other words, if the couplings between the qubit and the many-body system commute with the free Hamiltonian of the many-body system, the many-body system would remain in its initial state if the initial state is one of the eigenvalues of the free Hamiltonian H_m. Thus, the many-body system would make no effect on the qubit during the dynamics, and consequently the geometric phase acquired by the qubit could not signal the critical points of the many-body system.

III. GEOMETRIC PHASES INDUCED BY THE XY SPIN CHAIN

In this section, we will present an example to illustrate the claim. Taken a spin-chain described by the one-

dimensional XY model as the quantum many-body system, the Hamiltonian for the whole system (spin+chain) may be given by

$$H = H_{\frac{1}{2}} + H_m + H_i, \tag{13}$$

where

$$\begin{aligned} H_{\frac{1}{2}} &= \mu s^z, \\ H_m &= -2\sum_{l=1}^{N}((1+\gamma)s_l^x s_{l+1}^x + (1-\gamma)s_l^y s_{l+1}^y + \lambda s_l^z), \\ H_i &= 4g\sum_{l=1}^{N} s^z s_l^z. \end{aligned} \tag{14}$$

Here s denotes spin operator of the qubit which couples to the chain spins s_l $(l = 1, ..., N)$ located at the lattice site l. The spins in the chain are coupled to the qubit through a constant g. The time evolution operator $U(t)$ for the whole system may be written as $U(t) = \sum_{j=\uparrow,\downarrow} U_j(t)|j\rangle\langle j|$. It is easy to show that $U_j(t)(j = \uparrow, \downarrow)$ satisfy $i\hbar\frac{\partial}{\partial t}U_j(t) = H_j U_j(t)$ with $H_j = -\sum_{l=1}^{N}(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda_j\sigma_l^z)$, where $\lambda_j = \lambda \pm g$. If the auxiliary qubits are initially in state $|j\rangle$, the dynamics and statistical properties of the spin chain would be govern by H_j, it takes the same form as H_m but with modified field strengths λ_j. The Hamiltonian H_j can be diagonalized by a standard procedure[29] to be $H_j = \sum_k \omega_{j,k}(\eta_{j,k}^\dagger\eta_{j,k} - \frac{1}{2})$, where $\eta_{j,k}(\eta_{j,k}^\dagger)$ are the annihilation (creation) operators of the fermionic modes with frequency $\omega_{j,k} = \sqrt{\varepsilon_{j,k}^2 + \gamma^2\sin^2\frac{2\pi k}{N}}$, $\varepsilon_{j,k} = (\cos\frac{2\pi k}{N} - \lambda_j)$, $k = -N/2, -N/2+1, ..., N/2-1$. The fermionic operator $\eta_{j,k}$ was defined by the Bogoliubov transformation as, $\eta_{j,k} = d_k\cos\frac{\theta_{j,k}}{2} - id_{-k}^\dagger\sin\frac{\theta_{j,k}}{2}$, where $d_k = \frac{1}{\sqrt{N}}\sum_l a_l\exp(-i2\pi lk/N)$, and the mixing angle $\theta_{j,k}$ was defined by $\cos\theta_{j,k} = \varepsilon_{j,k}/\omega_{j,k}$. Fermionic operators a_l were connected with the spin operators by the Jordan-Wigner transformation via $a_l = (\prod_{m<l}\sigma_m^z)(\sigma_l^x + i\sigma_l^y)/2$. The operators $\eta_{j,k}$ parameterized by j clearly do not commute with each other, this will leads to nonzero geometric phase in the auxiliary qubits as shown later on. Before going on to calculate the reduced density matrix, we present a discussion on the diagonalization of H_j. For a chain with periodic boundary condition, i.e., $\sigma_1 = \sigma_N$, boundary terms $H_{boun} \sim [(a_N^\dagger a_1 + \gamma a_N a_1) + h.c.][exp(i\pi M) + 1]$ have to be taken into acount[29, 30]. In this paper, we would work with H_{boun} ignored, because we are interested in finding a link between the criticality of the chain and the entanglement in the auxiliary qubits. In fact, the boundary effect can be ignored provided $N \to \infty$, this is exactly the situation we consider in this paper.

Having given an initial product(separable) state of the total system, $|\psi(0)\rangle = |\psi_{\frac{1}{2}}(0)\rangle \otimes |\phi_m(0)\rangle$, we can obtain the reduced density matrix for the auxiliary qubits as $\rho_{\frac{1}{2}}(t) = \mathrm{Tr}_m[U(t)|\psi(0)\rangle\langle\psi(0)|U^\dagger(t)]$, it may be formally written in the form

$$\rho_{\frac{1}{2}}(t) = \sum_{i,j}\rho_{ij}(t)|i\rangle\langle j|. \tag{15}$$

A straightforward but somewhat tedious calculation shows that $\rho_{ij}(t) = \rho_{ij}(t=0)F_{ij}(t)$, with

$$\begin{aligned} F_{ij}(t) &= \prod_k e^{\frac{i}{2}(\omega_{i,k}-\omega_{j,k})t}\{1 - (1-e^{i\omega_{i,k}t})\sin^2\frac{\theta_k - \theta_{i,k}}{2} - (1-e^{-i\omega_{j,k}t})\sin^2\frac{\theta_k - \theta_{j,k}}{2} \\ &+ (1-e^{i\omega_{i,k}t})(1-e^{-i\omega_{j,k}t})[\sin\frac{\theta_k - \theta_{i,k}}{2}\sin\frac{\theta_k - \theta_{j,k}}{2}\cos\frac{\theta_{i,k} - \theta_{j,k}}{2}]\}, \end{aligned} \tag{16}$$

where $\cos\theta_k = \frac{\cos(2\pi k/N) - \lambda}{\sqrt{(\cos(2\pi k/N)-\lambda)^2 + \gamma^2\sin^2(2\pi k/N)}}$. To derive this result, the spin chain was assumed to be initially in the ground state of H_m.

With these expressions, we now turn to study the geometric phase of the open system. For an open system, the state in general is not pure and the evolution of the system is not unitary. For non-unitary evolution as shown in Eq.(16), the geometric phase can be calculated as follows. First, solve the eigenvalue problem for the reduced density matrix $\rho_{\frac{1}{2}}(t)$ and obtain its eigenvalues $\varepsilon_k(t)$ as well as the corresponding eigenvectors $|\psi_k(t)\rangle$; Secondly, substitute $\varepsilon_k(t)$ and $|\psi_k(t)\rangle$ into

$$\Phi_g = \arg(\sum_k\sqrt{\varepsilon_k(0)\varepsilon_k(T)}\langle\psi_k(0)|\psi_k(T)\rangle e^{-\int_0^T\langle\psi_k(t)|\partial/\partial t|\psi_k(t)\rangle dt}). \tag{17}$$

Here, Φ_g is the geometric phase for the system undergoing non-unitary evolution [31], T is the total evolution

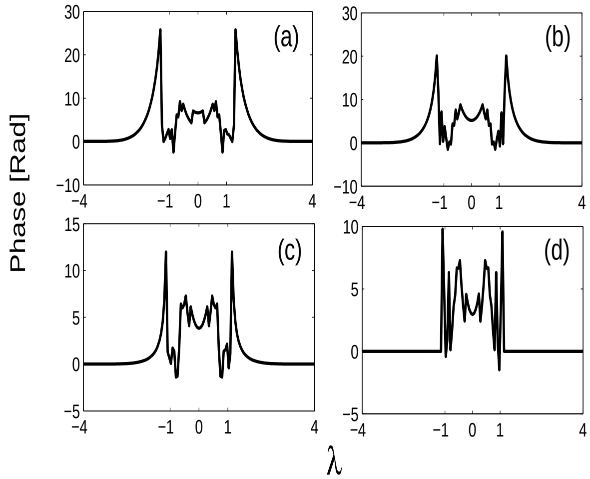

FIG. 1: The geometric phase as a function of λ. The figure was plotted for $N = 1245$ sites, $g = 0.1$ and (a)$\gamma = 0.2$,(b)$\gamma = 0.5$,(c)$\gamma = 0.8$,(d)$\gamma = 1$.

time. The geometric phase Eq. (17) is gauge invariant and can be reduced to the well-known results in the unitary evolution. It is experimentally testable. The geometric phase factor defined by Eq.(17) may be understood as a weighted sum over the phase factors pertaining to the eigenstates of the reduced density matrix, thus the detail of analytical expression for the geometric phase would depend on the digitalization of the reduced density matrix Eq.(16). The eigenvalues of the reduced density matrix are readily calculated

$$\varepsilon_{\pm}(t) = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\rho_{11}\rho_{22} + 4|\rho_{12}|^2}. \tag{18}$$

In order to calculate the geometric phase, we only need the eigenvalue $\varepsilon_+(t)$ and its corresponding eigenvector

$$|\varepsilon_+(t)\rangle = \cos\frac{\alpha}{2}e^{-i\phi}|\uparrow\rangle + \sin\frac{\alpha}{2}|\downarrow\rangle, \tag{19}$$

since $\varepsilon_-(0) = 0$. Here $\cos\alpha = \frac{(\rho_{11}-\rho_{22})}{2}/\sqrt{|\rho_{12}|^2 + \frac{1}{4}|\rho_{11} - \rho_{22}|^2}$, and $\tan\phi = \mathrm{I}m(\rho_{12})/\mathrm{R}e(\rho_{12})$, i.e., the ratio of the imaginary part to the real part of ρ_{12}. Having these results, we obtain the geometric phase,

$$\Phi_g = \arg(\sqrt{\varepsilon_+(0)\varepsilon_+(T)}\langle\varepsilon_+(0)|\varepsilon_+(T)\rangle e^{i\int_0^T \cos^2\frac{\alpha}{2}\dot{\phi}dt}). \tag{20}$$

We would like to notice that the geometric phase vanishes with $\gamma = 0$ because $F_{ij}(t) = 1$ at this point. $F_{ij}(t) = 1$ results in $\phi = 0$ in Eq.(20), this together with that $\langle\varepsilon_+(0)|\varepsilon_+(T)\rangle$ is real yield zero geometric phase at this critical point.

Now we turn to study the criticality in the transverse Ising model($\gamma = 1$ in the XY model). The ground state structure of this model change drastically as the parameter γ is varied. We first summarize the ground states of this model in the limits of $|\lambda| \to \infty$, $|\lambda| = 1$ and $\lambda = 0$. The ground state of the spin-chain approaches a product of spins pointing the positive/negative z direction in the $|\lambda| \to \infty$ limit, whereas the ground state in the limit $\lambda = 0$ is doubly degenerate under the global spin flip by $\prod_{l=1}^N \sigma_l^z$. At $|\lambda| = 1$, a fundamental transition in the ground state occurs, the symmetry under the global spin flip breaks at this point and the chain develops a nonzero magnetization $\langle\sigma_x\rangle \neq 0$ which increases with λ growing. The above mentioned properties of the ground state are reflected in the geometric phase as shown in figure 1-(d). In the limit $|\lambda| \to \infty$, $\theta_{j,k} = \theta_k = \pi/(-\pi)$, this results in $\Phi_g = 0$. In fact when $|\lambda| \geq 4$, Φ_g approaches zero very well. Around $|\lambda| \to 1$, the geometric phase changes drastically, this can be interpreted as the sensitivity of the spin-chain ground state to perturbations from the qubit-chain coupling at these points. The difference between $\gamma = 0$ and $\gamma = 1$ is that $F_{ij}(t) = 1$ for $\gamma = 0$, but it does not hold for $\gamma = 1$. This is the reason why the geometric phase takes different values at these critical points, as figure 1 shows. The ground state of the XY model is really complicated with many different regime of behavior[32], these are reflected in sharp changes in the geometric phase across the line $|\lambda| = 1$ regardless of γ (as shown in figure 1-(a),(b) and (c)), indicating the change in the ground state of the spin-chain from paramagnetic phase to the others. Instead of discussing the scaling behavior of the induced geometric phase, we consider the scaling property of $F_{ij}(t)$ given in Eq.(16) in the vicinity of the critical points. Noticing that $F_{ij}(t)$ is a function of $(\theta_k - \theta_{j,k})$ ($j =\uparrow, \downarrow$), the scaling behavior of $F_{ij}(t)$ may be characterized by $S_N^\lambda(\lambda,\gamma) = \sum_{k=1}^M (\frac{\partial\theta_k}{\partial\lambda})^2$, and $S_N^\gamma(\lambda,\gamma) = \sum_{k=1}^M (\frac{\partial\theta_k}{\partial\gamma})^2$[27]. It was shown that $S_N^\lambda(|\lambda| = 1,\gamma) \sim N^2/\gamma^2$ and $S_N^\lambda(|\lambda| \leq 1, \gamma = 0) \sim N^2$ in the vicinity of criticality for the one-dimensional XY Model.

IV. CONCLUSION

In conclusion, by discussing the geometric phase in the auxiliary qubit coupled to the many-body systems, the relation between the geometric phase induced in the qubit and the critical points of the many-body system was established. The induced geometric phase in the qubit change drastically at the critical points, this is due to the sensitivity of the many-body systum to parameter changes near its critical points. The relation not only provides an efficient theoretical tool to study quantum phase transitions, but also proposes a method to measure the critical points in experiments[33]. The limitation of this discussion is that the coupling between the qubit and the quantum system is assumed weak, and as we have shown, the geometric phase could not reflect the critical points of the quantum system that is initially in its ground state with qubit-system coupling satisfying $[H_m, H_i] = 0$.

This work was supported by NCET of M.O.E, and NSF of China under Grant No. 60578014.

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[33] The experiment to observe the prediction in this paper can be envisioned as follows. First let the qubit (a two-level system) transmit through a 50/50 beam splitter, the qubit is split and follows two different paths then. On one of the paths, the many-body system is placed and interact with the qubit. An interference patten can be observed when the two paths meet, this is similar to the Mach-Zehnder interferometer.